Hypothesis Testing

1. Define: (i)Optimal Test, (ii)Locally Best Test, (iii)Tests Under Restricted Alternatives, (iv)Similar Region and Neyman Structure, (v)Most Powerful Similar Region (MPSR) Test, (vi)Uniformly most Powerful Similar Region (UMPSR) Test
2. State and prove Neyman–Pearson lemma.
3. Given the frequency function:

$$f(x,\theta)=\begin{cases}\frac{1}{\theta}, & 0\le x\le\theta\\ 0, & elsewhere\end{cases}$$

And that you are testing the null hypothesis $H_0: \theta = 1$ against $H_1: \theta = 2$ by means of a single observed value of x. What would be the size of the type I and type II errors, if you chose the interval $(i) 0.5 \le x,\ (ii) 1 \le x \le 1.5$ as the critical region? Also obtain the power function of the test.

4. If $x \ge 1$ is the critical region for testing $H_0: \theta = 2$ against the alternative hypothesis $H_0: \theta = 1$, on the basis of single observation from the population,

$$f(x,\theta)=\theta\exp(-\theta x),\qquad 0\le x\le\infty$$

Obtain the values of type I and type II errors, also the power function of the test.

5. Let p be the probability that a coin will fall head in asingle toss in order to test $H_0: p = \frac{1}{2}$ against $H_1: p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of type I error and power of the test.
6. Let X have a p.d.f. of the form:

$$f(x,\theta)=\begin{cases}\frac{1}{\theta}e^{-\frac{x}{\theta}}, & 0<x<\infty,\theta>0\\ 0, & elsewhere\end{cases}$$

To test $H_0: \theta = 2$ against $H_1: \theta = 1$, use the random sample x_1, x_2 of size 2 and define a critical region: $W = \{(x_1, x_2): 9.5 \le x_1 + x_2\}$

Find: (i) Power of the test & (ii)Significance level of the test

7. Use the Neyman-Pearson lemma to obtain the region for the testing $\theta = \theta_0$ against $\theta = \theta_1 > \theta_0$ and $\theta = \theta_1 < \theta_0$, in the case of a normal population $N(\theta, \sigma^2)$, where σ^2 is known. Hence find the power of the test.
8. What are the methods of constructing test. Explain Union-Intersection test with an example.
9. Explain Intersection-Union test with an example.
10. Explain the relationship between Likelihood Ratio Test and Intersection-Union teast.
11. Explain: (i)Sobel's test, (ii)Lagrange Multiplier test or Score test and (iii)Armitage test.

Resampling

12. Explain Two sample test with an example .
13. Define ASN . How can we calculate ASN in resampling ?
14. A traditional hypothesis test for the mouse data might begin with the assumption that *F* and *G* are normal distributions with possibly different means $F = N(\mu_T, \sigma^2)$ $G = N(\mu_C, \sigma^2)$
    Construct the Hypothesis and calculate the ASN. If $\hat{\theta} = 30.63$ find the ASN and make a comment
15. What do you mean by Permutation test ? Write down the algorithm of two sample permutation test statistic.
16. Prove that all permutations of z's and y's are equally likely if F=G.
17. Explain bootstrap test statistic . write the algorithm of Bootstrap test statistic for testing F=G.
18. Explain the Relationship between the permutation test and the bootstrap.
19. Write the algorithm of Bootstrap test statistic for testing equality of means.
20. What is cross validation?
21. What are the general ideas of calibration.